

**Stochastic Analysis**  
**Ito Integrals and Stochastic Differential Equations with Jumps**

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In this text, we describe Ito integrals (stochastic integrals) and Ito formula as important tools in the theory of stochastic processes, and we describe a part of analysis of Markov processes with jumps by using stochastic differential equations (SDE's). (We assume the readers are well-known about basics of probability theory.)