Lévy Processes

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Contents

In this text we describe "additive processes", which are in a basic group of stochastic processes, and which have independent increments. Especially, we investigate "Lévy processes" in detail, which are time homogeneous additive processes, continuous in probability, and have first-order discontinuous sample paths, that is, they are right-continuous and have left-hand limits.

We will show the following: For every Lévy process, it has infinitely divisible distributions and their characteristic functions are given by the **Lévy-Khintchine representations**.

Also its sample path has the **Lévy-Ito decomposition**, which is a sum of Gaussian process with drift and a jump process.

Reference. SATO, Ken-ichi; "Kahou Katei" (in Japanese) as "Additive Processes", Kinokuniya (1990). This was rewritten in English and revised to "Lévy Processes and Infinitely Divisible Distributions", Cambridge (1999, 2002).

This text is based on the above text and the proofs are almost the same. However, the author tried to simplify and to make refinements in order to understand easily.